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Multi Depot Vehicle Routing Problem with Time Windows**  
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# *A Cooperative and Adaptive Variable Neighborhood Search for the Multi Depot Vehicle Routing Problem with Time Windows*

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## *Abstract*

*In this paper we propose two cooperation schemes to compose new parallel variants of the Variable Neighborhood Search (VNS). On the one hand, a coarse-grained cooperation scheme is introduced which is well suited for being enhanced with a solution warehouse to store and manage the so far best found solutions and a self-adapting mechanism for the most important search parameters. This makes an a priori parameter tuning obsolete. On the other hand, a fine-grained scheme was designed to reproduce the successful properties of the sequential VNS. In combination with the use of parallel exploration threads all of the best solutions and 11 out of 20 new best solutions for the Multi Depot Vehicle Routing Problem with Time Windows were found.*

*Keywords: Parallelization, Cooperation, Adaptation, Variable Neighborhood Search, Multi Depot Vehicle Routing Problem with Time Windows.*

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## **1 Introduction**

In recent years, cluster and grid architectures have become more and more popular. These architectures enable the design and development of cooperating algorithms to solve complex problems in the field of combinatorial optimization more efficiently than their sequential counterparts. The cooperation can take place between the same metaheuristic paradigms (e.g., Alba, 2005; Crainic and Toulouse, 2002), between different metaheuristics (e.g., Le Bouthillier and Crainic, 2005) or combinations of metaheuristics and mathematical programming (e.g., Fischetti and Lodi, 2003; Hansen, Mladenović, and Urosevic, 2006).

The aim of this paper is twofold: First, we propose a cooperative and adaptive algorithm based on the philosophy of the Variable Neighborhood Search (VNS). This metaheuristic described by Hansen and Mladenović (1999) is applied to solve Multi

Depot Vehicle Routing Problems with Time Windows. Second, in combination with the use of parallel exploration threads new best solutions were found. The sequential algorithm was published by Polacek, Hartl, Doerner, and Reimann (2004) and also applied to a real-world routing problem (Polacek, Doerner, Hartl, Kiechle, and Reimann, 2007). For the p-median problem, a cooperative implementation of the VNS was recently developed (e.g., Crainic, Gendreau, Hansen, and Mladenović, 2004; Garcia Lopez, Melian Batista, Moreno Perez, and Moreno Vega, 2002). Moreno Perez, Hansen, and Mladenović (2005) provide a survey of parallel VNS implementations.

In recent years, some papers on the parallelization of algorithms for solving the capacitated vehicle routing problem have been published (e.g., Jozefowicz, Semet, and Talbi, 1999, 2005; Ralphs, 2004). Jozefowicz, Semet, and Talbi (1999) devel-



oped a parallel Pareto genetic algorithm as well as a Pareto tabu search for a bi-objective VRP whereas Ralphs (2004) developed a parallel exact procedure based on branch and cut for the problem at hand. For the Vehicle Routing Problem with Time Windows Le Bouthillier and Crainic (2005) developed a cooperative parallel metaheuristic. Many applications were developed in the last few years in the broader field of parallel computing in transportation (e.g., Florian and Gendreau, 2001). In the book by Alba (2005) a recent and comprehensive overview of the different parallel metaheuristics can be found. To make the use of parallel metaheuristics accessible to a broad range of users, different libraries were developed by Alba and the MALLBA Group (2002) and Cahon, Melab, and Talbi (2004). Moreno Perez, Hansen, and Mladenović (2005) outline four different parallel VNS approaches. The first strategy analyzed by Garcia Lopez, Melian Batista, Moreno Perez, and Moreno Vega (2002) parallelizes the local search in the sequential VNS to get a balanced load among the processors and is denoted as Synchronous Parallel VNS (SPVNS). The second approach called Replicated Parallel VNS (RPVNS) and described by Crainic, Gendreau, Hansen, and Mladenović (2004) simply runs an independent VNS procedure on each processor. This non-cooperating parallelization is characterized by a multi start behavior. The same authors also report a more complex parallel variant denoted as Cooperative Neighborhood VNS (CNVNS) where several independent VNS processes cooperate by asynchronously exchanging information about the best solution identified so far. Communication takes place after a complete iteration through the set of neighborhoods. The last parallel strategy introduced is the Replicated Shaking VNS (RSVNS) proposed by Garcia Lopez, Melian Batista, Moreno Perez, and Moreno Vega (2002). RSVNS uses a synchronous cooperation mechanism where each worker processor generates one neighboring solution and applies the local search. In this paper we discuss different cooperation schemes and we propose an adaptive VNS where no a priori parameter tuning is necessary. First, from a technical point of view, it presents the first cooperative and adaptive implementation of a VNS for this problem and several design issues for cooperation and adaptation of the VNS algorithm are discussed. Second, from a problem oriented point of view, the computa-

tional results show that the approach is competitive with the sequential VNS implementation (Polacek, Hartl, Doerner, and Reimann, 2004) and the Tabu Search (TS) algorithm published in (Cordeau, Laporte, and Mercier, 2001, 2004), with respect to both solution quality and computation times. The parallelization strategy we use is an extension of the one implemented in CNVNS. The worker processes communicate exclusively with the master process which operates as the central memory. This allows an asynchronous cooperation of individual processes. In our proposed variants each worker has to search through a certain number of neighborhoods. However, compared to the CNVNS, in the fine-grained cooperation scheme this must not necessarily conclude the whole set of neighborhoods in one worker task. In the coarsegrained cooperation scheme, however, the number of iterations performed by each worker is vastly higher than the number of neighborhoods. This results in a more independent search via individual processes. The remainder of the paper is organized as follows: The routing problem is illustrated in Section 2 and the solution procedure of the sequential algorithm is discussed in Section 3. Section 4 reviews the main ideas of the cooperation and adaptation schemes and provides the details of the implementation and the design choices. Computational results are presented and discussed in Section 5. Section 6 concludes the paper with a resume of the applied approach.

## 2 Problem Description

The parallel VNS is applied to the Multi Depot Vehicle Routing Problem with Time Windows (MDVRPTW). It is a generalization of the well-known Vehicle Routing Problem with Time Windows (VRPTW) where instead of one depot, several depots with different locations and associated fleets have to be considered. The number of customers is denoted by  $n$  and the number of depots is denoted by  $m$ . Thus, the problem is defined on a complete graph  $G = (V, A)$ , where  $V = \{v_1, \dots, v_m, v_{m+1}, \dots, v_{m+n}\}$  is the vertex set and  $A = \{(v_i, v_j) : v_i, v_j \in V, i \neq j\}$  is the arc set. Vertices  $v_1$  to  $v_m$  correspond to  $m$  depots, while the vertices  $v_{m+1}$  to  $v_{m+n}$  represent  $n$  customers. Each vertex  $v_i \in V$  has several non-negative weights associated with it, namely, a demand  $d_i$ , a service time  $s_i$ , as well as an earliest  $e_i$  and latest  $l_i$  possi-



















